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Resonant Frequencies of an Electromagnetic Cavity in an Accelerated System of Reference*

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The resonant frequencies of an electromagnetic cavity in a nonpermanent gravitational field such as that provided by rotation is considered. The constitutive equations for accelerated macroscopic matter are developed and it is shown that the electromagnetic-field tensors are related by $H_{xt}=K_eF_{xt}$ and, $K_mH^{xy}=F^{xy}$, etc. For the special case of angular rotation about the z axis the constitutive equations are given in terms of noncovariant field vectors **B**, **H**, **D**, **E**. Using these constitutive equations Maxwell's equations are solved, and it is shown that the degeneracy between the axially degenerate modes, i.e., clockwise and counterclockwise traveling waves, is removed by the rotation. In vacuum due to the similarity between energy flux and momentum a shift in frequency of $\mathbf{\Omega}\cdot\mathbf{J}/h$ or a splitting between traveling wave modes of $2\mathbf{\Omega}\cdot\mathbf{J}/h$ is predicted. This led to a suggestion of the "Coriolis-Zeeman" effect for photons. Using either the above development or the energy density it is shown that the effect depends on the moment of the energy flux and is only proportional to the angular momentum. In the geometrical-optics region an index of refraction valid for accelerated macroscopic matter is developed and is applied to the square Fabry-Perot cavity rate gyroscope to yield the same result as either of the above methods. An experiment to show the validity of Minkowski's decomposition of the energy-momentum tensor is discussed.

INTRODUCTION

THE effect of acceleration on the resonant frequencies of an electromagnetic resonant structure or cavity is considered in this paper. Interference of light rays in a rotating system was suggested by Michelson as early as 1904 on the basis of the ether.[1] The observations of subsequent workers[2] confirmed the measurement of rotation by the interference technique and Michelson and Gale succeeded in measuring the angular rotation of the earth.[3] With the advent of the general theory of relativity it was shown that problems related to light transmission in accelerated frames of reference were more properly treated within the framework of general relativity. Adopting this viewpoint the author suggested that in rotating resonant-electromagnetic structures the degeneracy of the traveling wave modes about the axis of rotation would be removed by rotation and a beat frequency between the two modes of $2m\Omega$ would be observed.[4] This work was extended to optical masers[5] and confirmed by the experiments of Macek and Davis,[6] and Cheo and Heer.[7] It is the purpose of this paper to extend this earlier work.

In order to pursue the subject in a systematic manner the general principle of relativity that all systems of reference are equivalent with respect to the fundamental laws of physics is invoked. This requirement that the laws of nature be expressible in the form of equations which are form invariant or covariant is made possible by incorporating the metric $g_{\alpha\beta}$ into the physical laws. The $g_{\alpha\beta}$ are defined by the quadratic form

$$ds^2=g_{\alpha\beta}dx^\alpha dx^\beta. \tag{1}$$

For a system which is rotating with angular velocity Ω about the z axis and described by coordinates $x^1=x$, $x^2=y$, $x^3=z$, $x^4=ct$, the symmetric tensor $g_{\alpha\beta}$ has

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1 A. A. Michelson, Phil. Mag. **8**, 716 (1904).
2 G. Sagnac, Compt. Rend. **157**, 708 and 1410 (1913); F. Harress, dissertation, Jena, 1911 (unpublished); and B. Pogany, Ann. Physik **85**, 244 (1928).
3 A. A. Michelson and H. G. Gale, J. Astrophys. **61**, 1401 (1925).
4 C. V. Heer, Bull. Am. Phys. Soc. **6**, 58 (1961).
5 C. V. Heer, *Proceedings of the Third International Conference on Quantum Electronics* (Dunod Cie, Paris, and Columbia University Press, New York, 1963); A. H. Rosenthal, J. Opt. Soc. Am. **52**, 1143 (1962).
6 W. Macek and D. Davis, Appl. Phys. Letters **2**, 67 (1963).
7 P. K. Cheo and C. V. Heer (to be published).

the following nonzero covariant and contravariant components.

$$g_{xx}=g_{yy}=g_{zz}=1;\quad g_{xt}=g^{xt}=(-\Omega y/c);$$
$$g_{yt}=g^{yt}=(\Omega x/c);\quad g^{xx}=(1-\Omega^2y^2/c^2);$$
$$g^{yy}=(1-\Omega^2x^2/c^2);\quad g^{zz}=1;\tag{2}$$
$$g^{xy}=(\Omega^2xy/c);\quad g_{tt}=-[1-\Omega^2(x^2+y^2)/c^2];$$
$$g^{tt}=-1;\quad \det g\equiv g=-1.$$

Of the simple systems, i.e., constant linear velocity, linear acceleration, and angular rotation, only angular rotation does not permit the introduction of an instantaneous rest frame for the entire system and the metric cannot be placed in diagonal form. This aspect of the metric is considered in some detail by Möller.[8] It is also for this reason that effects of the order of v/c are observed in the rotating system. Thus, the experiment of Michelson can be explained by regarding the trajectory of a light ray as given by $ds^2=0$.

Maxwell's Equations

Following the procedure suggested in the Introduction, Maxwell's equations are written in a covariant form in an instantaneous rest frame, and by the rules of tensor algebra, remain valid in an arbitrary frame. For macroscopic matter Maxwell's equations in covariant form are

$$\frac{\partial F_{\alpha\beta}}{\partial x^\gamma}+\frac{\partial F_{\beta\gamma}}{\partial x^\alpha}+\frac{\partial F_{\gamma\alpha}}{\partial x^\beta}=0,$$

and

$$(-g)^{-1/2}\frac{\partial}{\partial x^\beta}[(-g)^{1/2}H^{\alpha\beta}]=j^\alpha.\tag{3}$$

In the subsequent discussion $g_{\alpha\beta}$ remains dimensionless, $F_{\alpha\beta}$ and $H^{\alpha\beta}$ have the dimensions of $(\text{energy})^{1/2}$, and the m.k.s. system of units is used. Although $F_{\alpha\beta}$ and $H^{\alpha\beta}$ can be written in terms of a polar vector and an axial vector in a covariant manner, a noncovariant notation[9] is found more convenient and consistent with conventional techniques. Nonconvariant vectors are defined as

$$\epsilon_0^{1/2}E_x=F_{xt};\quad \mu_0^{-1/2}B_x=\tfrac{1}{2}\delta_{xyz}F_{yz};\quad \mu_0^{1/2}J_x=j^x;$$
$$\epsilon_0^{-1/2}D_x=H^{tx};\quad \mu_0^{1/2}H_x=\tfrac{1}{2}\delta_{xyz}H^{yz};\quad \epsilon_0^{-1/2}\rho=j^t;\tag{4}$$

where δ_{xyz} is the three-dimensional Levi-Cevita symbol. Maxwell's equations take on their conventional appearance in this notation,

$$\operatorname{curl}\mathbf{E}+\partial\mathbf{B}/\partial t=0,\quad \operatorname{div}\mathbf{B}=0,$$
$$\operatorname{curl}\mathbf{H}-\partial\mathbf{D}/\partial t=\mathbf{J},\quad \operatorname{div}\mathbf{D}=\rho,\tag{5}$$

but the operations curl and div are defined only for Cartesian ordering of the differentiation.

[8] C. Möller, *The Theory of Relativity* (Oxford University Press, London, 1957).
[9] J. Plebanski, Phys. Rev. **118**, 1396 (1960).

Instantaneous Rest Frame

Using the rules of tensor algebra for two systems of coordinates which are related by

$$dX^\Phi=\bar{A}^\Phi{}_\alpha dx^\alpha,\qquad dx^\alpha=A^\alpha{}_\Phi dX^\Phi,\tag{6}$$
$$\bar{A}^\Phi{}_\alpha A^\alpha{}_\Psi=\delta^\Phi{}_\Psi,\qquad A^\alpha{}_\Psi\bar{A}^\Psi{}_\beta=\delta^\alpha{}_\beta,$$

the covariant and contravariant components of a tensor transform as

$$T_{\alpha\beta}=\bar{A}^\Phi{}_\alpha\bar{A}^\Psi{}_\beta T_{\Phi\Psi},\quad\text{and}\quad T^{\alpha\beta}=A^\alpha{}_\Phi A^\beta{}_\Psi T^{\Phi\Psi}.\tag{7}$$

Following the procedure discussed by Möller[8] an instantaneous rest frame is regarded as a system of inertia for which $G_{XX}=G_{YY}=G_{ZZ}=1$ and $G_{TT}=-1$. If (x,y,z,ct) and (X,Y,Z,cT) are regarded as instantaneously at rest with respect to each other, then

$$A^x{}_T=\bar{A}^X{}_t=0.\tag{8}$$

Since $g_{\alpha\beta}$ is a tensor, the components of the metric $g_{\alpha\beta}$ are related to those in the instantaneous rest frame by

$$g_{\alpha\beta}=\bar{A}^\Phi{}_\alpha\bar{A}^\Psi{}_\beta G_{\Phi\Psi}.\tag{9}$$

For subsequent reference,

$$g_{tx}=-\bar{A}^T{}_t\bar{A}^T{}_x,\quad\text{and}\quad g_{tt}=-\bar{A}^T{}_t\bar{A}^T{}_t.\tag{10}$$

Also in the remainder of this paper capital Greek or Latin indices refer to the instantaneous rest frame, small indices refer to the frame under consideration, and repeated Greek letters are summed over the four indices. Repeated Latin letters are summed over only the space indices.

CONSTITUTIVE EQUATIONS

For vacuum $H^{\alpha\beta}=F^{\alpha\beta}$, and the relationship between **B**, **H**, **E** and **D** follow immediately from the transformation $F^{\alpha\beta}=g^{\alpha\mu}g^{\beta\gamma}F_{\mu\gamma}$. Macroscopic matter requires further elaboration. A procedure similar to that used by Minkowski for introducing the constitutive equations in special relativity is used to find the constitutive equations in a nonpermanent gravitational field. For isotropic dielectric and paramagnetic substances the constitutive equations in an instantaneous rest frame have their usual form

$$D_X=K_e\epsilon_0E_X,\quad B_X=K_m\mu_0H_X;\quad J_X=\sigma E_X\tag{11}$$

in noncovariant notation. K_e is the dielectric constant, K_m the magnetic constant, and σ the electrical conductivity. In the instantaneous rest frame there is no distinction between covariant and contravariant components and these constitutive equations may be written as

$$H_{XT}=K_eF_{XT};\quad F_{YZ}=K_mH_{YZ};$$
$$j_X=(\mu_0/\epsilon_0)^{1/2}\sigma F_{XT};\quad j^T=\epsilon_0^{-1/2}\rho_I.\tag{12}$$

In any other frame of reference the components follow from the rules given by Eqs. (7) and (8). Thus

$$F_{xt}=\bar{A}^X{}_x\bar{A}^T{}_tF_{XT}\quad(\text{sum }X\text{ only}),\tag{13a}$$

and similar expressions follow for H_{xt}. A simple relationship between H_{xt} and F_{xt} follows, i.e.,

$$H_{xt}=\bar{A}^X{}_x\bar{A}^T{}_tH_{XT}=K_e\bar{A}^X{}_x\bar{A}^T{}_tF_{XT}=K_eF_{xt}. \quad (13b)$$

Following a similar procedure for F^{xy} and H^{xy}, the first two constitutive equations may be written in the simple form as

$$H_{xt}=K_eF_{xt} \quad \text{and} \quad K_mH^{xy}=F^{xy}. \quad (14a)$$

Noncovariant notation may be introduced by relating the covariant and contravariant components;

$$K_mH^{xy}=F^{xy}=g^{x\alpha}g^{y\beta}F_{\alpha\beta}, \quad \text{and} \quad K_eF_{xt}=H_{xt}=g_{x\alpha}g_{t\beta}H^{\alpha\beta}.$$

For the special example of rotation along the z axis, the introduction of the metric given by Eqs. (2) and the noncovariant field vectors yields the following constitutive equations. With $\mathbf{\Omega}=\Omega\mathbf{a}_z$ these expressions reduce to

$$\mathbf{B}+\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\times\left[\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\times\mathbf{B}\right]=K_m\mu_0\mathbf{H}+c^{-1}\left[\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\times\mathbf{E}\right],$$

$$\mathbf{D}+\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\times\left[\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\times\mathbf{D}\right]=K_e\epsilon_0\mathbf{E}-c^{-1}\left[\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\times\mathbf{H}\right]. \quad (14b)$$

For vacuum, $K_e=1=K_m$, these equations may be rearranged to read

$$\mathbf{B}=\mu_0\mathbf{H}+(\mu_0/\epsilon_0)^{1/2}[(\mathbf{\Omega}x\mathbf{r}/c)\times\mathbf{D}],$$
$$\mathbf{D}=\epsilon_0\mathbf{E}-(\epsilon_0/\mu_0)^{1/2}[(\mathbf{\Omega}x\mathbf{r}/c)\times\mathbf{B}]. \quad (14c)$$

Rotation has the effect of making even the vacuum appear anisotropic. This effect occurs in first order of Ω and corresponds to the type term introduced by Michelson for the description in terms of the "aether."

The source equation transforms as a four vector, and by the rules of tensor algebra

$$j_x=\bar{A}^X{}_xj_X+\bar{A}^T{}_xj_T, \quad \text{and} \quad j_t=\bar{A}^T{}_tj_T. \quad (15)$$

For macroscopic matter the constitutive equations in the instantaneous rest system are given by Eqs. (12). Substituting for j_X and j_t yields

$$j_x=(-g_{tt})^{-1/2}(\mu_0/\epsilon_0)^{1/2}\sigma F_{xt}-g_{tx}(-g_{tt})^{-1}j_t. \quad (16a)$$

Again using the metric to determine the components of j^x, the current and charge in the rotating system may be written as

$$\mathbf{J}=\sigma\left\{\mathbf{E}-\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\left[\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\cdot\mathbf{E}\right]\right\}\times[1-\Omega^2(x^2+y^2)/c^2]^{-1/2},$$

$$\rho=\left\{\sigma c^{-1}\left(\frac{\mathbf{\Omega}x\mathbf{r}}{c}\right)\cdot\mathbf{E}+\rho_I\right\}[1-\Omega^2(x^2+y^2)/c^2]^{-1/2}. \quad (16b)$$

Matter with a finite conductivity in the presence of an oscillating electric field will appear to have an induced oscillating current and charge. It is often desireable to separate the property of matter from the vacuum field. In the notation used in the paper this is expressed by

$$H^{\alpha\beta}=F^{\alpha\beta}-M^{\alpha\beta}$$

and noncovariant vectors **M** and **P** for the magnetization and polarization are defined as

$$\mu_0^{1/2}M_x=\tfrac{1}{2}\delta_{xyz}M^{xy}, \quad \text{and} \quad \epsilon_0^{-1/2}P_x=M^{xt}. \quad (17)$$

With these definitions, Maxwell's equations may be written in terms of **B** and **E** if **J** is replaced by

$$\mathbf{J}\to\mathbf{J}+\text{curl}\mathbf{M}+\partial\mathbf{P}/\partial t. \quad (18)$$

RESONANT FREQUENCY OF A ROTATING CAVITY

The approximate frequency of oscillation of a resonant electromagnetic cavity or structure is found by a method used by Slater[10] for microwave cavities. His method is modified to include traveling waves and it is necessary to use complex wave functions. A set of orthonormal functions $\mathbf{E}_a$ and $\mathbf{H}_a$ which are solutions of the vector Helmholtz equation and which satisfy the boundary conditions $\mathbf{n}\times\mathbf{E}_a=0$ and $\mathbf{n}\cdot\mathbf{H}_a=0$ on a perfectly conducting surface S and $\mathbf{n}\times\mathbf{H}_a=0$ and $\mathbf{n}\cdot\mathbf{E}_a=0$ on an insulating surface S' are introduced. These vectors are related by $-ik_a\mathbf{H}_a=\text{curl}\mathbf{E}_a$ and $ik_a\mathbf{E}_a=\text{curl}\mathbf{H}_a$. $k_a\mathbf{F}_a=\text{grad}\psi_a$ for the scalar potential. Orthogonality is chosen such that $\int dv\mathbf{E}_a\cdot\mathbf{E}_b{}^*=\delta(a=b)$, etc., and an arbitrary vector may be expanded in terms of this basic set, i.e.,

$$\mathbf{E}(\mathbf{r},t(=\sum_a\mathbf{E}_a\int dv\mathbf{E}\cdot\mathbf{E}_a{}^*+\sum_\beta\mathbf{F}_\beta\int dv\mathbf{E}\cdot\mathbf{F}_\beta{}^*, \quad (19)$$

where the volume integrals are the time-dependent coefficients. Expanding Maxwell's equations in this manner yields

$$-ik_a\int dv\mathbf{E}\cdot\mathbf{E}_a{}^*+\frac{d}{dt}\int dv\mathbf{B}\cdot\mathbf{H}_a{}^*=-\int_S dA(\mathbf{n}\times\mathbf{E})\cdot\mathbf{H}_a{}^*, \quad (20a)$$

$$+ik_a\int dv\mathbf{H}\cdot\mathbf{H}_a{}^*-\frac{d}{dt}\int dv\mathbf{D}\cdot\mathbf{E}_a{}^*=\int dv\mathbf{J}\cdot\mathbf{E}_a{}^*-\int_{S'}dA(\mathbf{n}\times\mathbf{H})\cdot\mathbf{E}_a{}^*, \quad (20b)$$

$$\int dv\mathbf{D}\cdot\text{grad}\psi_a{}^*+\int dv\rho\psi_a{}^*=0. \quad (20c)$$

This expansion in normal modes implies that the element

[10] J. C. Slater, *Microwave Electronics* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1950).

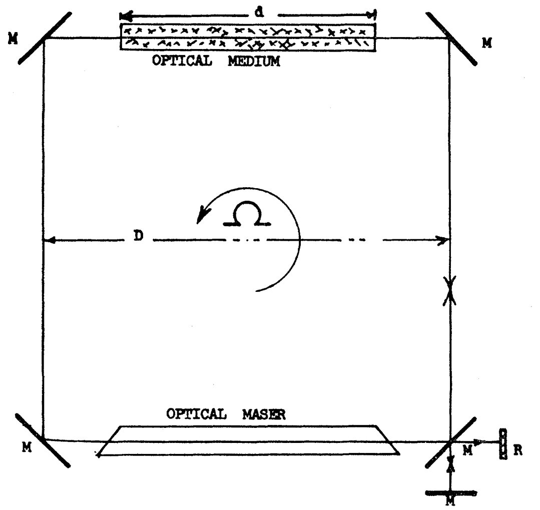

FIG. 1. Four mirrors M form the square Fabry-Perot cavity with diameter D. Optical media of length d and index of refraction $(K_eK_m)^{1/2}$ shifts the splitting in resonant frequency of the axially degenerate modes which is caused by the angular rotation Ω from the vacuum value given by Eq. (25) to that given by Eq. (26). Beats are observed with the photodetector R.

of volume is $dxdydz$ and the surface elements $dxdy$, etc. This is not true in the rotating system and a correction of the $\mathcal{O}(\Omega^2)$ is implied in the wave functions. The above equations are correct to $\mathcal{O}(\Omega^2)$ and should yield the resonant frequency to $\mathcal{O}(\Omega^2)$. Omitting the source terms on the right and using the constitutive equations to eliminate **B** and **D**, the remaining equations may be solved to yield the resonant frequency. If $\nu_a{}^0$ is the resonant frequency, and $\mathbf{E}_a$ and $\mathbf{H}_a$ the normal modes for $\Omega=0$, the resonant frequency correct to $\mathcal{O}(\Omega)$ becomes

$$\nu_a=\nu_a{}^0\bigg\{1+\tfrac{1}{2}(K_eK_mc^2)^{-1/2}\boldsymbol{\Omega}\cdot\int dv[\mathbf{r}x(\mathbf{E}_a{}^*\times\mathbf{H}_a+\mathbf{E}_a\times\mathbf{H}_a{}^*)]\bigg\}\,, \quad (21)$$

where as emphasized earlier $\boldsymbol{\Omega}$ is along the z axis. If two modes a and a' are axially degenerate for $\Omega=0$, then the frequency splitting between the two modes is given by

$$\Delta\nu/\nu_a=(K_eK_mc^2)^{-1/2}\boldsymbol{\Omega}\cdot\int dv[\mathbf{r}x(\mathbf{E}_a\times\mathbf{H}_a{}^*+\mathbf{E}_a{}^*\times\mathbf{H}_a)]+\mathcal{O}(\Omega^2)\,. \quad (22)$$

The cylindrical microwave cavity with rotation Ω along the z or symmetry axis provides an excellent example. In the TE_{mnp} mode the zero-order wave functions are of the form

$$\mathbf{E}_a=A_a\{\hat{a}_z\times\text{grad}[e^{im\varphi}Z_m(\alpha_{mn}\rho)]\}\sin\pi pz/z_0\,. \quad (23)$$

A direct evaluation of the integral for vacuum yields

$$\Delta\omega=2\pi\Delta\nu=2m\Omega\,, \quad (24)$$

and the rotation removes the twofold degeneracy of the axial mode.

A system of interest as a sensitive rate gyroscope[5–7] is the square Fabry-Perot cavity with confocal or flat mirrors. If $\mathbf{E}_a$ is an almost plane wave with a finite cross section, and if the perpendicular axis of the cavity makes angle γ with the z axis of rotation, integration over the beam volume yields a frequency splitting of

$$\Delta\nu/\nu_a=(\Omega D/c)\cos\gamma \quad (25)$$

for vacuum, D is the diameter of the square. Other configurations may be treated in a similar manner. If for convenience $\gamma=0$ and one arm of the Fabry-Perot contains matter as shown in Fig. 1 with an index of refraction $(K_eK_m)^{1/2}$ and of length d, then

$$\Delta\nu/\nu_a=(\Omega D/c)\{1-(d/4D)[1-(K_eK_m)^{-1/2}]\} \quad (26)$$

is the required correction for the matter.

Plane Waves

In the limit of short wavelengths or of the validity of geometrical optics the invariance of the phase $d\varphi=k_\alpha dx^\alpha$ and the invariance of the scalar $k_\alpha k^\alpha$ can be used to determine the index of refraction as measured by an observer in a nonpermanent gravitational field. Fermat's principle can be applied to determine the path. Using the transformation between k_t and k_T, the scalar invariant becomes

$$k_\Phi k^\Phi=k_\alpha k^\alpha=(\omega/c)^2(K_eK_m-1)(-g_{tt})^{-1}\,, \quad (27)$$

and is of course the usual null vector for vacuum. If $d\sigma$ is a line element along the path of the ray and n^a is the contravariant component of the ray direction, then

$$n^a=dx^a/d\sigma=k^a c\omega^{-1}(-g_{tt}/K_eK_m)^{1/2} \quad (28)$$

follows from Eq. (27) and the various relationships for the $g_{\alpha\beta}$.[11] Further manipulation yields for the phase

$$d\varphi=k_\alpha dx^\alpha=\omega c^{-1}\{[(-K_eK_m/g_{tt})^{1/2}-g_{at}n^a/g_{tt}]d\sigma-cdt\}\,. \quad (29)$$

If the quantity inside the square brackets is regarded as the index of refraction μ,

$$\mu=(-K_eK_m/g_{tt})^{1/2}-g_{at}n^a/g_{tt}\,, \quad (30)$$

then the path follows the variation

$$\delta\int\mu d\sigma=0\,. \quad (31)$$

Equation (31) is quite general and is Fermat's principle applicable for macroscopic matter and nonpermanent gravitational fields. If the principle of equivalence is invoked and for the $g_{at}=0$, the bending of a light ray in a macroscopic medium and in a permanent gravita-

[11] See L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1951), p. 280, for a similar development for vacuum.

tional field described by g_{tt} is readily obtained. ($-g_{tt} = 1+2\chi/c^2$ where χ is the gravitational potential.) It is interesting to note that the bending by a gravitational field is not separable from that bending by matter. For the special example a system rotating about the z axis and described by the metric given by Eq. (2), μ becomes

$$\mu = (K_e K_m)^{1/2} + c^{-1}(\mathbf{\Omega}\times\mathbf{r})\cdot\hat{n} + \mathcal{O}(\Omega^2), \tag{32}$$

where $\hat{n}$ is a unit normal in the direction of the ray. Macroscopic matter does not effect the term depending on g_{at} and the term of $\mathcal{O}(\Omega)$ in the index of refraction depends on the vacuum properties only. If this index of refraction is used for the square Fabry-Perot cavity, or cavities in which geometrical optics may be used with some validity, the shift in resonant frequency is the same as indicated by Eqs. (25) and (26).[12]

CORIOLIS-ZEEMAN EFFECT FOR PHOTONS

The removal of the twofold degeneracy of the axial mode of the cylindrical cavity discussed earlier yields an angular frequency of

$$\omega_{mnp} = \omega_{mnp}^0 \pm m\Omega \quad (m=0, 1, 2, \cdots). \tag{33}$$

If the photon concept is used and $\mathbf{J}$ is the orbital angular momentum of the photon, the above equation may be written as

$$\hbar\omega = \hbar\omega^0 + J_z\Omega, \tag{34}$$

and the correction is proportional to $\mathbf{\Omega}\cdot\mathbf{J}$. For vacuum, Eq. (22) is consistent with the concept of angular momentum. In fact, Eq. (25) is readily obtained by noting that

$$\Delta\nu/\nu = 2\mathbf{\Omega}\cdot\mathbf{J}/h\nu = (\Omega D/c)\cos\gamma, \tag{35}$$

where $J_z = (h\nu/c)(D/2)\cos\gamma$ is the projection of the orbital angular momentum $\mathbf{J}$ along the z axis. The observer in the rotating system would find the resonant frequency dependent on the angular momentum of the photon and would regard the level splitting as a "Coriolis-Zeeman" effect for the photon. This concept is valid only for vacuum. The right side of Eq. (22) is the expression for the moment of energy flux rather than angular momentum. This distinction between energy flux and momentum is apparent only in the presence of matter or in $\mathcal{O}(\Omega^2)$.

If the energy-momentum tensor is defined as

$$S_\alpha{}^\beta = F_{\alpha\mu}H^{\beta\mu} - \tfrac{1}{4}\delta_\alpha{}^\beta F_{\mu\nu}H^{\mu\nu}, \tag{36}$$

whose covariant divergence yields the four-force $f_\alpha = F_{\alpha\beta}j^\beta$, the various components in the noncovariant notation are

$$\begin{aligned} S_t{}^x &= -c^{-1}(\mathbf{E}\times\mathbf{H})_x, \quad S_x{}^t = c(\mathbf{D}\times\mathbf{B})_x, \\ S_t{}^t &= -\tfrac{1}{2}(\mathbf{E}\cdot\mathbf{D}+\mathbf{B}\cdot\mathbf{H}). \end{aligned} \tag{37}$$

In the instantaneous rest frame $S_T{}^T$ is the energy density, $S_T{}^X$ the energy flux, and $S_X{}^T$ the momentum, and these quantities have the same interpretation in other systems of coordinates. The energy-momentum tensor in the instantaneous rest frame and the frame of observation are related by

$$S_\Phi{}^\Psi = A^\alpha{}_\Phi \bar{A}^\Psi{}_\beta S_\alpha{}^\beta, \tag{38}$$

and the energy density may be written as

$$S_T{}^T = S_t{}^t + (g_{tx}/g_{tt})S_t{}^x. \tag{39}$$

For the stationary system which is being considered the $g_{\alpha\beta}$ do not depend on the local time variable t and for short periods of time in which losses may be ignored, solutions of Maxwell's equations which are independent of the time variable may be obtained. The observer in the moving system interprets the integral of $S_t{}^t$ over the volume of the cavity or radiation field at time t as a constant of motion for the system. If a single photon is present the photon frequency is measured as

$$\nu = h^{-1}\int dv\, S_t{}^t = h^{-1}\int dv[S_T{}^T - (g_{tx}/g_{tt})S_t{}^x]. \tag{40}$$

Considered as a traveling wave the integral over $S_T{}^T$ is independent of the sense of rotation and the beat frequency caused by the interference of clockwise (c.w.) and counter-clockwise (c.c.w.) photons at a detector attached to the moving system is

$$\begin{aligned} \Delta\nu &= 2h^{-1}\int dv[g_{tx}/g_{tt}]S_t{}^x \\ &= 2h^{-1}c^{-2}\mathbf{\Omega}\cdot\int dv[\mathbf{r}x(\mathbf{E}\times\mathbf{H})] + \mathcal{O}(\Omega^2). \end{aligned} \tag{41}$$

This is the same as the earlier expression given by Eq. (22) when $\mathbf{E}$ and $\mathbf{H}$ are replaced by zero-order orthonormal $\mathbf{E}_a$ and $\mathbf{H}_a$ wave functions. The dependence on the projection of the moment of the energy flux rather than the angular momentum is immediately apparent.

MINKOWSKI'S ENERGY-MOMENTUM TENSOR

The lack of symmetry in the Minkowski energy-momentum tensor between the energy flux $S_T{}^\Phi$ and the momentum $S_\Phi{}^T$ in the presence of macroscopic matter has been discussed frequently in the literature.[13]

The energy-momentum tensor is symmetric in an inertia frame in the absence of matter and becomes

[12] Although an experiment with matter waves does not seem possible at this time, it is interesting to note that using the same formalism with the Klein-Gordon equation a phase difference between the c.w. and c.c.w. paths is obtained in which the Compton wavelength $\lambda = h/mc$ replaces the optical wavelength. The phase shift is $8\pi D^2\Omega/\lambda c$ in an experiment similar to that used by Michelson (Ref. 3). Interference depends on the Compton rather than the de Broglie wavelength (Ref. 4).

[13] See Ref. 8, p. 204 for a discussion and references to related discussions.

nonsymmetric in the presence of matter or nonpermanent gravitational fields. A first-order nonsymmetry appears in the presence of rotation. Minkowski's theory requires that transparent matter not exchange energy with the electromagnetic field. It is shown in the earlier developments in this paper that the frequency shift in first order of Ω depends on the moment of the energy flux and also that the change in index of refraction in the geometrical optic region is independent of the presence of matter to $\mathcal{O}(\Omega)$. These same results follow with the Minkowski energy-momentum tensor given by Eq. (36). Thus an observation of the beat frequency in accord with Eq. (26) in the presence of matter would support the Minkowski formulation. This effect should be observable in the square Fabry-Perot cavity with flat mirrors for which the mode of operation is simple. Since the presence of the maser material allows the cavity to oscillate at both frequencies and the beat frequency is sensitive to rotation only, the shift in the beat frequency is measureable. Combining Eqs. (25) and (26) for ν_a coinciding with a maser line, the fractional shift is

$$(\Delta\nu_M/\Delta\nu_0)=1-(d/4D)[1-(K_eK_m)^{-1/2}], \quad (42)$$

where $\Delta\nu_M$ is the beat frequency with matter and $\Delta\nu_0$ with vacuum. If a large portion of the maser is filled with matter an effect as large as 30% can be expected. Unfortunately terms of $\mathcal{O}(\Omega^2)$ cannot be tested by present experimental apparatus. Applying the principle of equivalence it is the terms of $\mathcal{O}(\Omega^2)$ which are equivalent to the effects of a permanent gravitational field. Although not emphasized in the development, the equations developed in this paper may be applied in a permanent gravitational field by invoking the principle of equivalence.

EXCITATION OF CAVITY MODES

Using the form invariance of the fundamental laws of physics and the concept of the instantaneous rest frame, the constitutive equations for macroscopic matter in an accelerated system have been developed and used with Maxwell's equations to discuss the resonant frequencies of electromagnetic cavities. The deformation of matter by the acceleration forces has been neglected. Since the deformation of matter by the acceleration would change the shape of the cavity and the physical constants of enclosed matter, the small shifts suggested would be difficult to observe. For axially degenerate modes the deformations affect both c.w. and c.c.w. traveling waves in the same manner and the splitting between modes given by Eq. (22) is almost independent of the deformations. The splitting between modes given by Eq. (22) is valid over the entire region of the electromagnetic spectrum for which cavity modes may be defined. Thus, in the microwave region a reasonable size cavity would have a value of $m\sim 10$, while in the optical region a value of $m>10^6$ is possible. For a closed microwave cavity the normal modes may be determined in a standard manner and wall losses introduced by discussing the decay of stored energy in terms of the Q. Since the beat frequency would be of the order of cycles per second for a rotation rate of 1 rad per sec, a Q of the order of 10^9 would be required to resolve the splitting between the c.w. and c.c.w. modes. Such a Q is almost possible with superconducting cavity walls. In the optical region the Fabry-Perot cavity shown in Fig. 1 may be discussed in a similar manner. A system with dimensions of the order of 1 m has the Q limited to 10^8 by diffraction[14] and reflection losses and a beat note of the order of kilocycles is not resolved.

In order to enhance the apparent Q of the system a maser material in the appropriate frequency range may be used.[5–7] The volume source or sink term on the right-hand side of Eq. (20b) may be due to either the polarization, magnetization, or conductivity as shown by Eq. (18). If the maser media has a narrow linewidth described by $\epsilon''(\nu)$ or $\chi''(\nu)$,[15] either property may be discussed in terms of a generalized negative resistance $\sigma(\nu)$ which is proportional to it. In this linear approximation the response of the c.w. mode depends on the value of $\sigma(\nu)$ at the frequency of the c.w. mode. A similar argument follows for the c.c.w. mode, and both modes will be excited for a maser linewidth greater than the splitting. Oscillation will occur for both modes in the linear approximation. If the polarization depends on the electric field in a nonlinear manner, the question of coupling between these closely spaced modes needs further consideration.[16] It may be possible to study these nonlinearities by using the square Fabry-Perot cavity as a precision spectrometer system.

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